The Alternating Series Test

1. Consider the series $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n}$. We call this the alternating harmonic series.
   (a) Why can't we use the p-test or either of the comparison tests to determine the convergence or divergence of this series?
   (b) Write out the first few terms of the sequence of partial sums. Make a guess about the convergence or divergence of the series.
   (c) Prove the convergence or divergence of the series with an appropriate test.
2. Consider the series $\sum_{n=2}^{\infty}(-1)^{n-1}\frac{3n}{n^2-2}$.
   (a) Why can't we use the comparison tests to determine the convergence or divergence of this series?
   (b) Make a guess about the convergence or divergence of the series.
   (c) Both the numerator and denominator of $b_n = \frac{3n}{n^2-2}$ are increasing. Use the derivative of $f(x) = \frac{3x}{x^2-2}$ to determine whether $\{b_n\}$ is increasing, decreasing, or neither.
   (d) Prove the convergence or divergence of the series with an appropriate test.
3. Determine whether each of the following series converge or diverge.
   (a) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{3n^2}{9n^2+9}$
   (b) $\sum_{n=3}^{\infty}(-1)^{n}\frac{n}{\sqrt{n^3+2}}$
   (c) $\sum_{n=1}^{\infty}(-1)^{n}\cos\left(\frac{\pi}{n}\right)$
   (d) $\sum_{n=2}^{\infty}(-1)^{n}\sin\left(\frac{\pi}{n}\right)$
   (e) $\sum_{m=1}^{\infty}(-1)^{m}\cos(\pi m)$

4. For which values of p does $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n^p}$ converge?